5.3 Definite Integrals and Antiderivatives

Objective

SWBAT use properties of definite integrals, average value of a function, mean value theorem for definite integrals, and connect differential and integral calculus.

Properties of Definite Integrals

In defining the integral as a limit of sums, we moved from left to right across the interval [a,b]. What about integrating the opposite direction? The integral would become ____________ - again a limit of sums, but this time the delta x would be _______________ as the x-values decreased from b to a. This would change the signs of all the terms in each Riemann sum, and ultimately the sign of the definite integral. This suggests the rule

Since the original definition did not apply to integrating backwards over an interval, we can treat this rule as a logical extension of the definition. Although [a,a] is technically not an interval, another logical extension of the definition is that _________________________.

These are the first two rules that are also given below. The others are inherited from rules that hold for Riemann sums. However, the limit step required to ___________ that these rules hold in the limit (as the norms of the partitions tend to zero) places their mathematical verification beyond the scope of this course. They should make sense nonetheless.

Example 1 *Using the Rules for Definite Integrals*

Suppose

$$\int_{-1}^{1} f(x)\,dx = 5, \int_{1}^{4} f(x)\,dx = -2, \int_{-1}^{1} h(x)\,dx = 7$$

Find each of the following integrals, if possible.

a) Integral of f(x) on interval [4,1].
b) Integral of f(x) on interval [-1,4].
c) Twice the integral of f(x) plus three times the integral of h(x) on the [-1,1].
d) Integral of f(x) on interval [0,1].
e) Integral of h(x) on interval [-2,2].
f) Integral of the sum of f(x) and h(x) on the interval [-1,4].

Example 2 *Finding Bounds for an Integral*

Show that the value of $\int_0^1 \sqrt{1 + \cos x}\,dx$ is less than 3/2.

Average Value of a Function

The _______________ of n numbers is the ______ of the numbers divided by n. How would we define the average value of an arbitrary function f over a closed interval [a,b]? As there are infinitely many values to consider, adding them and then dividing by _____ is not an option.

Consider, then what happens if we take a large sample of n number from regular subintervals of the interval [a,b]. One way would be to use Riemann sums, if you want to see why, check the top of page 287.

What we end up seeing is that when we consider this average process as n approaches infinity, we find it has a ___________, namely ______________ time the integral of f over [a,b].

Average (Mean) Value

If f is integrable on [a,b], its average (mean) value on [a,b] is

Example 3 *Applying the Definition*

Find the average value of f(x) = 4 - x^2 on [0,3]. Does f actually take on this value at some point in the given interval?

Mean Value Theorem for Definite Integrals

It was no mere coincidence that the function in the previous example took on its average value at some point in the interval. The statement that a continuous function on a closed interval _____________ assumes its average value at least once in the interval is known as the __________ for Definite Integrals.

Theorem 3 *The Mean Value Theorem for Definite Integrals*
If f is continuous on [a,b], then at some point c in [a,b],

Connecting Differential and Integral Calculus

As you have asked in the previous section, how do we evaluate definite integrals when there is no known area formula to apply? For example, the integral of sin x from 0 to pi equals 2. Would Newton and Leibniz know this fact and how? They did know that ________________ of infinitely small quantities, as they put it, could be used to get velocity functions from positions function , and the _______ of infinitely thin "rectangle areas" could be used to get position functions _______ velocity functions. In some way there had to be a connection between these two seemingly different processes. Newton and Leibniz were able to picture that connections and this led them to the __________________________________.

We can conclude that the derivative with respect to x of the integral of f from a to x is simply f. Specifically.

This means that the integral is an _____________________ of f, a fact we can exploit in the following way. If F is any antiderivative of f , then

for some constant C. Setting x in this equation equal to a gives,

Putting it all together,

The implications of the previous last equation were enormous for the discoverers of calculus. It meant that they could evaluate the definite integral of f from a to any number x simply by computing ________________, where F is any ______________________ of f.

Example 4 *Finding an Integral Using Antiderivatives*

Find $\int_0^\pi \sin x \, dx$ using the formula $\int_a^x f(t)\, dt = F(x) - F(a)$.

Homework

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